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MATHEMATICS FOR THE PHYSIOLOGIST AND PHYSICIAN.*

BY H. B. WILLIAMS.

It seems to have been very generally assumed that a student intending to enter upon the study of medicine needs little if any preparation in mathematics beyond arithmetic and perhaps a little algebra and geometry. I am informed that students in Columbia College intending to enter the College of Physicians and Surgeons, have been definitely advised by a member of the medical faculty that they would have little if any need for mathematical training and that they would better spend their time in preparation along other lines of study. The physician who gave this advice has been very successful in the practice of medicine and is well known as an educator. I have heard similar views expressed by another well-known and successful New York physician. I presume that the views of these eminent medical men are fairly representative of the opinion of the majority of the medical profession today and inasmuch as the student who contemplates a career in medicine naturally—and very properly—seeks the advice of older men in the profession, it is not surprising that these students come to their professional studies with a mathematical training which is meager in the extreme. This attitude of indifference, sometimes approaching hostility toward the teaching of mathematics to prospective medical students, seems to have sprung in part from the circumstance that many of the shining lights in the medical profession have attained their position in the firmament without having felt the need of more powerful mathematical aids than the average high-school courses afford. We are still hardly beyond the period when similar arguments were supposed to prove that a college education is a superfluous ornament for men who expect to spend their lives in what for want of a more specific term is usually called "business." I believe I shall be able to show that although a large measure of usefulness may be attained in the field of medicine by the non-mathematically trained physician, nevertheless the inclusion of such training in his preparation for life will enable him to extend his efforts into a wider field of useful endeavor.

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At the outset I would invite your attention to the dual aspect of medicine: it is both a science and an art. One studies the science and the art and practises the latter. The best physicians admittedly are those who while practising their art keep constantly abreast of the advances in the science of medicine and, so far as opportunity and ability permit, contribute by their own efforts to that advancement. To determine what constitutes that necessary and sufficient mathematical preparation which will enable a student to develop into a scientific physician well equipped to cope with the problems of his profession, or into an original investigator in any one of the fundamental medical sciences, we must inquire into the nature of these sciences themselves.

Of these the first to be developed was anatomy, the science of structure, including histology, which deals with structure of such smallness as to require the assistance of the microscope for its study. This department of scientific medicine is almost purely descriptive and since it offers fewer opportunities for the application of mathematics than some of the others, we shall not enter into further discussion of it. Physiology is the science of function; it deals with the manner in which the living structure carries on those activities which distinguish it from non-living things. No short definition is likely to be complete and satisfactory, but a statement that physiology is the physics and chemistry of living things will direct attention to the phases of the subject which are most pertinent to the present discussion. Pathology is the science of abnormal structure and abnormal function and comprises in the requirements for its successful cultivation all that is included in anatomy and histology as well as all that is included in physiology. Bacteriology, the botany of the most minute plants, many of which have been shown to play an important part in the production of pathological conditions, is in medical schools closely allied with pathology. Biochemistry is a special department of chemistry which concerns itself with those chemical substances and reactions which are associated with the life processes of plants and animals, either as constituents of the living organism, foodstuffs out of which it provides material for growth and repair .of waste, or by-products of its life activities. The substances with which the biochemist deals are for the most part of very complex character, and his training in the fundamental branches of his science can scarcely be too thorough.

From this brief statement it will appear without further elaboration that the student of medicine or the investigator in the medical sciences requires training in both physics and chemistry. How much of this training is requisite? If one aspires to do research in either physics or chemistry he will scarcely need argument to convince him that thorough familiarity with the existing state of his chosen science is indispensable. In the cultivation of the medical sciences he will be obliged to investigate phenomena by the same means employed by the physicist and the chemist, but the phenomena will be of a far higher order of complexity. If he is to spend his time profitably, avoiding waste of it and the commission of gross errors, his training in the fundamental science of physics and chemistry, including physical chemistry, should be most thorough. It may be illuminating to point out that the great advances in the medical sciences have not been made by men with scanty training. To mention but one notable example from the field of physical physiology, the work of Helmholtz constitutes probably the largest contribution to this branch of the science ever made by a single individual. Helmholtz was thoroughly trained in the mathematical and physical lore of his day and outside the medical profession he is probably better known for his contributions to physics than for his physiological investigations. It is of more than passing interest to note that most of the work he did in the field of physiology has stood the test of time. To this man, trained originally as a physician and holding chairs in the medical sciences in three famous universities before transferring his activities entirely to the field of physics, we are indebted among other things for our knowledge that the nerve impulse travels at a finite speed, a speed which he measured; for the theory of quality or timbre of musical sounds; for the invention of the opthalmoscope and opthalmometer and for what to the best of my knowledge remains to this day the most complete treaties on physiological optics. Who shall say that equally fruitful fields do not still await cultivation by scientific physicians who have the forethought and industry to lay in early life the necessary foundation in the fundamental sciences?

The ideal training then for men who are to enter upon careers of investigation in the medical sciences would be such as to make them thoroughly trained theoretical and experimental chemists and physicists with all of the mathematical preparation which this involves, this in addition to the special training in anatomy, zoology, botany and related subjects without which physical and chemical knowledge cannot be applied to the problems of living things. Manifestly the brevity of human life makes this an ideal, the attainment of which is impossible. What then shall we advise? We find that most students who intend to enter eventually upon the study of medicine or upon a professional career in one of the medical sciences elect during their courses in college to study such subjects as general biology, zoology, histology, comparative anatomy, botany, perhaps a little chemistry, usually no more physics and mathematics than is required by the institution they happen to attend for the particular degree to which their course will lead. I have no fault whatever to find with these subjects, all of them are excellent as preparation for a lifeworn to be spent in medicine or in the medical sciences; but are they the subjects to be chosen at the time when students generally elect to take them? I venture to think they are not, at least not for students whose purpose is to prepare for the study of medicine. These students in their professional courses will be given much that would be included in the general biological studies I have mentioned. They will have acquired after the first two years of the medical course such training as would enable them by individual reading to become familiar with any of the subjects mentioned above and should get in a good medical school laboratory training at least as good as they would get in college courses in the biological subjects. In all of their work they will constantly need the fundamental subjects, physics and chemistry, and before they can make satisfactory progress in the study of either of these it will be necessary for them to secure adequate mathematical preparation. I believe that this foundation should be laid before biological subjects are elected at all.

I realize that there are many physicians who are content to practise the art of medicine without paying much more attention to the progress of the science than is absolutely necessary to enable them to maintain their position among their co-practitioners. They do a necessary work and they have their reward. I am reasonably sure that they would all do their work much better if they were to continue throughout their professional careers to cultivate the science as well as the art. Certainly if a member of my own family were in need of medical attention it would not be to the mere artisans among my colleagues that I should turn for assistance. For the man who is content to practise medicine as a blacksmith shoes horses, I presume all the mathematics necessary would be that simple arithmetic which would enable him to avoid errors in writing prescriptions and in sending out his monthly bills. It is not for these that I make my plea for more extensive preparation in physics, chemistry and such mathematics as is necessary for the profitable study of these. My concern is rather for the student of vision and aspirations who wishes to fit himself for whatever work in his chosen profession opportunity may later throw in his way. To such I would say, secure if possible a full four years of academic training and let it include a large measure of physics, chemistry and physical chemistry, with the mathematical preparation which it is well recognized must accompany these subjects. Then if you find it necessary in later life to branch out into regions of endeavor where you have had no special training, you will be provided with the essential weapons with which any problem in natural science must be attacked, will be able to extend your education without the assistance of a mentor along whatever lines opportunity or inclination indicates.

Perhaps I may be permitted to offer an illustration of the sort of thing which happens all too frequently when physicians with scanty training in the essentials undertake to do original work. A young man working on a problem in chemical physiology had undertaken to make a determination of hippuric acid. After weighing his product he wished to check its purity by determination of the melting point. Finding the melting point several degrees too high, he made repeated attempts to purify his product, but each time the melting point was the same. It was found that the melting point apparatus he was using involved a considerable stem correction, a matter of which he had never even heard. On applying the correction it appeared that his melting point was very nearly that for pure hippuric acid, and on changing the apparatus for one in which the stem correction was eliminated, his melting point was in agreement with that of other observers for pure hippuric acid with all the precision that could reasonably have been expected. This illustration does not directly involve mathematics, but had this young man received the practical training in physics and chemistry for which I have asked, he would probably have known how to determine melting points before starting original research or at least his training would have been such as to enable him to suspect the cause of discrepancy.

If required to say just what mathematics I consider most important for the scientific physician or the physiologist, I should specify as a minimum, algebra, to include summation of infinite series and the criteria of convergency and divergency, plane and solid geometry, plane trigonometry, the elements of analytical geometry, and at least an introduction to the concepts of the differential and integral calculus. If he has no more than this he will be able to read with at least some hope of understanding many scientific papers which would otherwise remain insoluble riddles. In his work as a practising physician he may never use any part of this training. It is for his use in connection with the underlying sciences during his medical training, in his subsequent scientific reading and in his original research work if he does it, that this preparation will bear fruit. If he can spare time he will make no mistake to include some analytical trigonometry, a thorough course in the calculus and in differential equations, vector analysis, the use of hyperbolic functions, theory of errors, theory of probabilities and the theory of determinants.

It is most unlikely that the average student fitting for the study of medicine will ever be induced to cover such a course of preparation in mathematics as this, probably for the *average* student the minimum would be all that would be desirable, but the student may at least be advised that he will be well repaid for the time and effort spent in acquiring whatever mathematical training he secures rather than be told that all mathematical study is for him a waste of time. For the student who contemplates a life devoted in the main to investigation either in physiology, pathology or any of the special branches of these large subjects, I would most earnestly advise that the minimum I have mentioned above be by all means secured. At the same time I would urge upon his teachers the advisability of instructing him from the start in the employment of his mathematics as an implement of research. For the student who wishes to use his mathematics principally on problems in physics and physical chemistry, it seems to me it would be much more appropriate for his instructor in mathematics to make use of physical rather than geometrical illustrations wherever possible. Geometry antedates modern physics by many years, geometrical illustrations have become classical and there is no objection to them for students of pure mathematics, but the student of natural science, especially the one inclined toward biological investigations, is apt to need some stimulus in the way of apparent practical application of his mathematical lore in order to tide him over that often uninteresting period where acquisition must precede application.

I believe I have sufficiently emphasized the need for mathematics in connection with the acquisition and use of physics and chemistry. It may be asked whether there are direct applications of mathematics to the problems of the physician and physiologists in his own particular sphere of activity. Strictly speaking I should say not, for careful analysis will show that nearly every problem to which the physician would apply mathematical methods is either a chemical or physical problem in medicine. However, one may mention the possibility of applying the theory of probabilities to medical statistics and of the use of graphic methods to represent biological phenomena. In connection with the latter there is the interesting possibility of fitting equations to the curves expressing these phenomena. I could give abundant examples of the indirect us of mathematics in connection with problems arising in the physiological laboratory. I need only mention the necessity for the physiologist to develop the theory of such instruments as he uses in order to ascertain in what way he may modify them to adapt them the better to a particular investigation. I might give as an instance the development by Einthoven* of the string galvanometer, an instrument widely used both by physicians and physiologists. The essential features of this unique instrument were deduced by Professor Einthoven entirely as a result of mathematical investigations. It may not be known to all of you that there are connected with all life processes, electrical phenomena the study of which has already yielded results of great interest. It is with a view to the successful pursuit of studies along these lines that I have mentioned the hyperbolic trigonometry as desirable. Vector analysis will prove a great convenience in this field of work and probably in others, not to mention the advantages to be derived from a knowledge of it in the study of the fundamental sciences, physics and chemistry. The determination of maxima and minima is an application of the calculus which is frequently used in connection with the re-designing of instruments or in determining how best to work with a particular apparatus. The mere mention of ophthalmology suggests geometrical optics, and one might go on multiplying particular instances.

In conclusion I should like to take this opportunity to bring to the attention of a group of men and women interested in the generality of science, the fact that there is at the present time a lack of adequately trained men in physiology. The younger men have not been entering this field of work in recent years in any considerable numbers. It is a field which offers much in the way of opportunity for original investigation to men of proper training. I believe I have already indicated what I mean by proper training. It has always been true that the best work in physiology has been accomplished by men with a broad foundation in the underlying natural sciences. In the future we shall need more than ever men of the very highest training to carry on physiological investigations. I believe the time is about over when research workers in this field will be recruited in any considerable number from the ranks of the medical profession. I believe also that it is in the best interests of science, general as well as medical, that physiology be regarded as a pure science, to be cultivated as such, not as a mere handmaid of medicine to be studied in medical schools alone. The establishment of physiology as a well-recognized department of graduate university work would undoubtedly help to bring its problems and opportunities to the attention of young men of attainments along the lines of physics and chemistry, and it seems to me that future progress depends largely upon the entrance into this arena of young men trained along the lines I have indicated.

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* W. Einthoven, "Ein neues Galvanometer, Annalen der Physik," fourth series, Volume 12, 1903.

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